Hard Photon Production Rate of a Quark-Gluon Plasma at Finite Quark Chemical Potential*

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Abstract

We compute the photon production rate of a quark-gluon plasma (QGP) at finite quark chemical potential μ using the Braaten-Pisarski method, thus continuing the work of Kapusta, Lichard, and Seibert who did the calculation for $\mu = 0$.

The thermal production of hard photons in ultrarelativistic heavy ion collisions has been proposed as a possible signature for the formation of a quark-gluon plasma (QGP) [1]. Since the mean free path of photons in the fireball is much larger than its dimensions [2], photons provide a direct probe of the fireball. So far the production rate of hard photons has been considered at finite temperature but vanishing quark chemical potential mainly. In order to include medium effects consistently the Braaten-Pisarski resummation technique [3] has been applied to this problem [4,5].

Assuming the formation of a QGP already at AGS and SPS energies, however, a finite quark chemical potential has to be considered [6], and even at RHIC energies the quark chemical potential μ may not be negligible as indicated by RQMD simulation ($\mu \approx 1 - 2\,T$)

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[7]. At given energy density estimates based on lowest order perturbation theory indicate a strong suppression of the photon production at non-vanishing μ as compared to the case $\mu = 0$ [8]. The aim of the present work is the improvement of these estimates by applying the Braaten-Pisarski method generalized to finite chemical potential [9].

We follow the calculation of Kapusta, Lichard, and Seibert [4], and describe the changes that have to be made for a nonzero quark chemical potential. For brevity, we do not demonstrate the intermediate steps of the computation where they would be analogous to ref. [4].

To leading order the hard photon production rate is derived by using the Braaten-Yuan prescription [10] resulting in a decomposition into a soft part, which is treated using the resummed propagators of Braaten and Pisarski, and a hard part containing only bare propagators. For this purpose a parameter k_c is introduced, separating the soft from the hard momenta of the intermediate quark. Demanding $gT \ll k_c \ll T$ and assuming the weak coupling limit, the final result is independent of the separation scale k_c.

The soft part can be obtained from the imaginary part of the self energy of a photon propagating through a QGP. The contributing diagrams to the photon self energy are shown in fig. 1. There quark lines with blobs represent effective quark propagators [11], however for the $\mu \neq 0$–case with a modified quark mass [9,12] of

$$m_q^2 = \frac{g^2}{6}\left(T^2 + \frac{\mu^2}{\pi^2}\right) \quad . \tag{1}$$

Our result for the soft contribution to the production rate of a hard photon with energy E and momentum $\boldsymbol{p}$ is

$$\begin{aligned} 2E\frac{dR^{\mathrm{soft}}}{d^3p} &= -\frac{2}{(2\pi)^3} g^{\mu\nu}\, Im\ \Pi^{\mathrm{ret}}_{\mu\nu} \frac{1}{e^{E/T}-1} \\ &= \frac{5\alpha\alpha_s e^{-E/T}}{9\pi^2}\left(T^2 + \frac{\mu^2}{\pi^2}\right) \ln\left(\frac{k_c^2}{2m_q^2}\right) \quad . \end{aligned} \tag{2}$$

This differs from the result obtained in ref. [4] only in that (a) the quark mass m_q depends now on the chemical potential and (b) the factor in front of the logarithm is modified since it originates from a factor m_q^2. (In deriving eq. (2) we assumed a hyperbolic cut-off

$k_c^2 > \boldsymbol{k}^2 - \omega^2 > 0$ in accordance to the hard part (6) following ref. [4], where ω is the energy and $\boldsymbol{k}$ the momentum of the spacelike intermediate soft quark. Thus ω and $|\boldsymbol{k}|$ need not to be soft individually. However, closer investigations show that for the expression under the loop integral in $\Pi_{\mu\nu}^{ret}$, approximations are legitimate that assume ω as well as $|\boldsymbol{k}|$ to be soft, since it is this region where the main contribution to the loop integral comes from.)

The easiest way to calculate the hard part is starting from the scattering matrix elements shown in fig. 2. They are related to the photon self energy of fig. 1 by cutting the latter [13]. From QCD Feynman rules, we find the squared matrix elements to be

$$\Sigma|\mathcal{M}|^2 = \frac{2^9 \cdot 5}{9} \pi^2 \alpha \alpha_s \frac{u^2 + t^2}{ut} \tag{3}$$

for the annihilation process, and

$$\Sigma|\mathcal{M}|^2 = -\frac{2^9 \cdot 5}{9} \pi^2 \alpha \alpha_s \frac{s^2 + t^2}{st} \tag{4}$$

for each of the two Compton processes for quarks and antiquarks. The symbol Σ indicates that these matrix elements are already summed over spins, colors, and two flavors (u and d). The letters s, t, and u denote the Mandelstam variables. In each diagram, all legs but the outgoing photon belong to thermalized particles. The photon is not thermalized since it has a large mean free path. For each of the three possible processes we may compute the hard part of the photon production rate as [4]

$$2E\frac{dR^{\mathrm{hard}}}{d^3p} = \frac{1}{(2\pi)^8} \int \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \frac{d^3p_3}{2E_3} \delta^4(P_1 + P_2 - P_3 - K) n_1(E_1) n_2(E_2)(1 \pm n_3(E_3)) \Sigma|\mathcal{M}|^2 \tag{5}$$

where $n_{1,2,3}$ are Bose or Fermi distribution functions, respectively; the plus sign is for the annihilation process and the minus for the two Compton processes. Still following ref. [4], we may rewrite this equation as

$$2E\frac{dR^{\mathrm{hard}}}{d^3p} = \frac{1}{8(2\pi)^7 E} \int\limits_{2k_c^2}^{\infty} ds \int\limits_{-s+k_c^2}^{-k_c^2} dt \, \Sigma|\mathcal{M}|^2 \int\limits_{\mathrm{IR}^2} dE_1 \, dE_2 \frac{\Theta(P(E_1, E_2)) n_1 n_2 (1 \pm n_3)}{\sqrt{P(E_1, E_2)}} \tag{6}$$

with the polynomial $P(E_1, E_2) = -(tE_1 + (s+t)E_2)^2 + 2Es((s+t)E_2 - tE_1) - s^2E^2 + s^2t + st^2$ and the step function Θ.

For the $\mu = 0$–case, it is a reasonable approximation to use Boltzmann distribution functions for n_1 and n_2 instead of the full Fermi/Bose functions. In this way, all integrations may be performed analytically, and the result

$$2E\frac{dR}{d^3p} = \frac{5}{9}\frac{\alpha\alpha_s T^2 e^{-E/T}}{\pi^2}\Bigg(\underbrace{\frac{2}{3}\ln\left(\frac{4ET}{k_c^2}\right) - 1.43}_{\text{annihilation}} + \underbrace{\frac{1}{3}\ln\left(\frac{4ET}{k_c^2}\right) + 0.015}_{\text{Compton}}\Bigg) \qquad (7)$$

fits perfectly together with the soft part: the sum of both is independent of k_c.

Actually, the Boltzmann approximation (for n_1 and n_2) is better than it should be, considered that the photon contribution from the Compton processes is significantly underestimated (up to 30%) and the annihilation contribution is overestimated. Surprisingly, both errors cancel each other, up to an error of about 10% in the final result for those values of T, E, and g that are interesting in practice.

The total photon production rate for $\mu = 0$, computed in the Boltzmann approximation (for n_1, n_2), is

$$2E\frac{dR}{d^3p} = \frac{5}{9}\frac{T^2\alpha\alpha_s e^{-E/T}}{\pi^2}\ln\left(\frac{2.91E}{g^2T}\right) \quad , \qquad (8)$$

which also follows from the photon damping rate by the principle of detailed balance [2].

Dumitru et al. [8] use the Boltzmann approximation also in their computation of the photon rate at $\mu \neq 0$, however they obtain a hard part which does not match onto the analytically known soft part, (2). The cited result also contains a term $\sim \mathrm{Ei}\left((4E\mu - k_c^2)/4ET\right)$ which runs over a branch point at finite μ.

Our approach to the problem is the following: since we cannot evaluate the integrals in (6) containing the exact distribution functions analytically, we employ numerical methods. We rewrite (6) in a form suitable for Gauss quadrature ($E_+ := E_1 + E_2$):

$$
\begin{aligned}
2E\frac{dR^{\rm hard}}{d^3p} = -\frac{5\alpha\alpha_s}{18\pi^5 E} e^{-E/T-k_c^2/2ET} & \underbrace{\int\limits_{2k_c^2}^{\infty} ds\, e^{-(s-2k_c^2)/4ET}}_{\text{Laguerre}} \quad \frac{1}{s}\underbrace{\int\limits_{-s+k_c^2}^{-k_c^2}}_{\text{Legendre}} dt\, |\mathcal{M}(s,t)|^2 \times \\
& \times \underbrace{\int\limits_{E+s/4E}^{\infty} dE_+\, e^{-(E_+-E-s/4E)/T}}_{\text{Laguerre}} \frac{1}{1\mp e^{-(E_+-E-\mu_3)/T}} \times \\
& \times \underbrace{\int\limits_{E_2^-}^{E_2^+} \frac{dE_2}{\sqrt{P_1(E_+,E_2)}}}_{\text{Chebyshev}} \frac{1}{e^{-\mu_1/T} \pm e^{-(E_+-E_2)/T}} \frac{1}{e^{-\mu_2/T}+e^{-E_2/T}} \quad .
\end{aligned}
\tag{9}
$$

In this compact notation, the upper signs and $\mu_1 = -\mu_2 = \mu$, $\mu_3 = 0$ apply to the annihilation process; the Compton processes require the lower signs and $\mu_1 = 0$, $\mu_2 = \mu_3 = \pm\mu$, where the two results for $+\mu$ and $-\mu$ have to be added in order to take antiquarks as well as quarks into consideration. The polynomial $P_1(E_+, E_2)$ is just $P(E_+ - E_2, E_2)/s^2$; thus it has the leading coefficient -1 and gives under the square root a perfect weight for a Gauss-Chebyshev quadrature in E_2. The E_+–integral, as well as the s–integral, is done numerically by a Gauss-Laguerre quadrature; the necessary exponential weight function arises naturally in the expression. The t–integral, having no appropriate weight function, is performed via Gauss-Legendre quadrature. Finally, the s–integral is done by a Gauss-Laguerre quadrature, as suggested again by the naturally arising weight function.

Unfortunately, each of the four integrands is highly peaked at the ends of the integration interval, due to singularities in or near the domain of integration. This means the integral value is dominated by contributions coming from comparably small regions, and the quadrature problem is ill-conditioned. In order to cure the problem, we subtract from the integrand in each step of the fourfold integration those contributions that stem from poles inside or slightly outside the integration region. Those parts are integrated out separately in an analytical way; the remaining numerical integrals are much easier to compute, since the integrands are no longer varying strongly. This way, we need only about 20 points for each quadrature, and still obtain numerical results for the hard part that are precise up to

an maximal error of 0.2%.

In fig. 3 k_c is varied over almost six orders of magnitude, leaving the total photon rate fixed within numerical error. (In order to test the cancellation of k_c, expected to occur in the weak coupling limit, we adopt a value of $g = 0.01$.) Even in regions where $gT \ll k_c \ll T$ is by no means true, the sum of soft and hard part is perfectly constant, while the soft part alone varies over a wide range and even becomes negative. This is a nice verification of the Braaten-Yuan method for $\mu \neq 0$ [9,10]. From (2) and the independence on k_c we know that the final result has to assume the form

$$2E\frac{dR}{d^3p} \; = \; \frac{5\alpha\alpha_s e^{-E/T}}{9\pi^2}\left(T^2+\frac{\mu^2}{\pi^2}\right)\left(\ln\frac{2.91ET}{g^2(T^2+\mu^2/\pi^2)}+G\right) \quad , \tag{10}$$

where the dimensionless quantity G follows from the hard part. As a result of the numerical calculation of the hard part, discussed above, it turns out that G depends only on μ/T. Within a 3% error in the rate, G depends very weakly on E/T. However, G is nicely independent of T, as a dimensionless quantity should be. We found that G can be fitted to a good approximation by the phenomenological formula $G = \ln(1 + \mu^2/\pi^2T^2)$, thus leading to the compact expression

$$2E\frac{dR}{d^3p} \; = \; \frac{5\alpha\alpha_s e^{-E/T}}{9\pi^2}\left(T^2+\frac{\mu^2}{\pi^2}\right)\ln\left(\frac{2.91E}{g^2T}\right) \quad . \tag{11}$$

This pocket formula reproduces the exact, numerically calculated result within an error of 3% for $|\mu/T| \leq 1$. A larger μ/T requires a one-parameter fit for G. We found that $G = \ln(1 + 0.139\mu^2/T^2)$, inserted in eq. (10), leads to a phenomenological formula for the rate that is precise on the 3%-level even for much larger $|\mu/T|$. We emphasize that the error of 3% is not due to the numerical evaluation of the hard part but to the assumption that G is solely dependent on μ/T.

We are now able to extrapolate this formula to realistic values of g and discuss the photon spectrum of the QGP. For photon energies E above about $3T$ the logarithm in (11) is positive, indicating the validity of the extrapolation to realistic values of the coupling constant [14]. Fig.4 shows how the spectra are dominated by the exponential decrease with

temperature. We draw the conclusion that a measurement of the photon spectrum may serve only or mainly to find out the temperature of the plasma; in order to measure the chemical potential by a photon experiment, one would have to measure the overall coefficient of the exponential, which is in an experimentally obtained spectrum influenced by many other factors not yet under control, like the size and duration of the plasma phase. However, if one has information on the energy density ϵ of the plasma, T and μ are related to each other by an equation of state like the one quoted in ref. [8]:

$$\epsilon = \left(\frac{37\pi^2}{30} - \frac{11\pi\alpha_s}{3}\right) T^4 + 3\left(1 - \frac{2\alpha_s}{\pi}\right) T^2\mu^2 + \frac{3}{2\pi^2}\left(1 - \frac{2\alpha_s}{\pi}\right)\mu^4 + (0.2\,GeV)^4 \quad . \tag{12}$$

If T is made dependent on μ in this fashion [8], the resulting photon spectra are strongly dependent on μ. At RHIC, one expects a maximum energy density of about $\epsilon = 5\,GeV/\,fm^3$. In fig. 5, we show the corresponding photon spectra. The temperatures of the five curves decrease with rising μ from $0.27\,GeV$ to about $0.22\,GeV$, whereas μ/T varies from 0 to about 1.8. The photon suppression at finite chemical potential and fixed energy density observed in ref. [8] is therefore an indirect phenomenon, caused by the reduction of the temperature.

In conclusion, at $\mu = 0$, our result (11) conincides with the one in ref. [4]. The Boltzmann approximation used in there seems to work fine.

At $\mu \neq 0$, Bose and Fermi distributions have to be used in order to match the soft contribution onto the hard one according to the Braaten-Yuan prescription providing a consistent result to leading order. Hence the hard part of the photon production rate can only be determined numerically. We were able to demonstrate that the final result is independent of the arbitrary separation parameter k_c and obtained the simple formula (11). Our graphs of the photon spectrum have a strong similarity to the one of Dumitru et al. [8], mainly because they all are dominated by the single factor $e^{-E/T}$.

We did not consider pre-equilibrium effects so far. The equilibrium distribution functions used in our calculation are not quite applicable, since the plasma is probably never completely equilibrated. For a phenomenological treatment of a pre-equilibrium plasma, fugacity factors may be employed [15]. This will be dealt with in a future publication.

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